Capacity and Conductivity of Body Tissues at Ultrahigh Frequencies

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Summary—Dielectric constant and specific resistance are reported for a variety of body tissues throughout the frequency range from 200 to 1,000 mc. The results are analyzed and explained by the cellular structure of tissue, the electrical properties of tissue electrolytes, and tissue protein content. Other results specify the temperature dependence of the electrical constants of tissue material. The temperature coefficients vary with frequency and are in agreement with theoretical expectation. A short description of measuring technique and of tissue sample thermostat is given.

INTRODUCTION

It has been stated previously1,2 that for various reasons, uhf-radiation diathermy may be most effective when operating in the frequency range around 500 mc. In discussions of the frequency dependence of the depth of penetration of uhf-radiation1 and of the relative development of heat in subcutaneous fat, as compared to that in muscle,2,3 it was assumed that the electrical properties of muscle and other tissues with high water content are similar to those of blood. This assumption was based on an analytical argument and necessary since only blood measurements had been conducted in the frequency range from 100 to 1,000 mc.4,5 It is intended to present in this paper:

1. Data of the electrical properties of various tissues in order to close the gap which exists between results which have been obtained by other investigators either below 100 mc,6 or above 1,000 mc.4,5 We further want to check the validity of the assumption concerning the similarity of the electrical properties of blood and tissues with high water content.

2. Temperature coefficients of the electrical properties of tissues in the frequency range mentioned above, to permit application of the measured data to other temperature levels of interest.

3. Theoretical analysis of the frequency dependence of electrical properties of tissue in terms of their structure and molecular content.

MEASURING TECHNIQUE

The measurements were carried out with a double wire system operating on a resonant principle as previously described.1,10 The use of an "open wire" system combines the advantages of low cost of construction with the possibility of keeping the biological sample under constant observation and simplicity of temperature control. The use of the resonant technique avoids disadvantages, which may be extremely disturbing with open systems such as oscillations of the system against third conductors (ground) and sensitivity against movement of personnel operating the transmission line. Furthermore, it reduces undesirable effects caused by harmonics. The biological sample (thickness d) surrounds one small section of the line and is loaded with a λ/4 section to provide infinite load to its terminals. The input impedance of the sample may be related to standing wave ratio \( W = V_{\text{max}}/V_{\text{min}} \), and displacement of the wave pattern ρ, which occurs when the sample holder is loaded. Both quantities, \( W \) and \( ρ \), are determined with the measuring line in front of the sample holder. The input impedance, as observed by the measuring line, is furthermore related to the dielectric properties of the material in the sample holder. By combination, we can eliminate the input impedance value and express the electrical constants of the material in the sample holder in terms of \( W \) and \( ρ \) as follows:11

\[
\begin{align*}
\epsilon &= \frac{\lambda}{2\pi d} \tan \frac{2\pi}{\lambda} \left( W^2 - 1 \right) \\
\rho &= \frac{120\pi d}{W} \left( W^2 + \tan^2 \frac{2\pi}{\lambda} \right) \left( \lambda, d \right)
\end{align*}
\]

(\(\varepsilon\) dielectric constant relative to air, \(\rho\) specific resistance in Ohm cm). The formulas are good approximations for small sample thickness values\(^{12}\) and a discussion of higher terms in a series development of an accurate solution of the problem has convinced us that they are sufficient for the present purpose. Details concerning derivation, as well as errors involved, elimination of the effect of the sample holder on the field, and a complete discussion of the errors involved with this technique will be given elsewhere.\(^{13}\)

The thermostat which was used for the variation of the sample temperature is shown in Fig. 1. The tissue material is introduced in thin slices, of 1 to 2 mm thickness, between the two center plates of polystyrene.

Slight pressure is applied by the plates to assure that the tissue sample thickness is identical with the thickness of the spacer rings and center pieces which keep the plates at a defined distance. The picture presents a side view with the two conductors of the double wire system appearing as one. Polystyrene rings are attached to the sample holder at a distance sufficiently great to avoid any effect on the field distribution around the wires. At the ends of the rings thin sheets of polystyrene (0.05 mm thick) are fastened with rubber bands. They have a center hole of about 1 1/2 inches and do not influence the field. Air enters through inlets in the bottom of the arrangement, and escapes through the center holes in the plastic sheets, thereby reducing the possibility that the high thermal conductivity of the conductors may cause a substantial decrease in temperature in the immediate vicinity of the center. The temperature of the air is at present regulated with a simple condenser arrangement, utilizing compressed air, which is available in the laboratory, and hot water flowing around the coils of the condenser. The temperature is regulated by variation in air and water flow or by variation of the water temperature circulating through the condenser. The arrangement has the great advantage that it combines effective performance with simplicity of construction. It has no measurable effect on the electrical field, which is essential in avoiding complications in the mathematical evaluation of the measurements.\(^{14}\)

The curves in Fig. 2 are self-explanatory and show that temperature stability is reached within one half to one hour. The temperature through the sample agrees within about 0.50°C with the temperature of the air, as measured with a thermometer. Another test was performed by measuring the electrical properties of electrolytic solution whose electrical resistivity at low frequencies was found to be 154 ohm cm at 27°C. The temperature coefficient of this solution is known from tables and permits calculation of its low frequency resistivity at 38°C, yielding a value of 131 ohm cm. Application of polar theory\(^{15}\) makes it possible to determine analytically the total frequency dependence of the resistivity of the saline solution. The characteristic frequency values for water necessary in this calculation are taken from Conner and Smyth,\(^{16}\) and supported by evaluation of water measurements as carried out by Herrick, Jelatis, and Lee\(^{7}\) and us.\(^{16}\) The result is given in Fig. 3 and fits with the experimental results within a few per cent. The dielectric constant is frequency independent according to Debye's theory up to about 1,000 mc. The difference

\[\text{Fig. 1—Side view of sample holder with thermostat.}\]

\[\text{Fig. 2—Temperature of thermostat and sample as function of time. These are two different experiments carried out with different air temperature.}\]
in dielectric constant between 27° C. and 38° C. is in our case 5 in agreement with earlier observations on distilled water.16,17 The deviation of the experimental points from the theoretical curves is explained by an error estimate according to which dielectric constants are accurate within about 2 per cent and resistance value accurate within 5 per cent.

![Figure 3](image)

**Fig. 3**—Dielectric constant and specific resistance of NaCl solution at 27° and 38° C.

### Results

The Tables 1 and 2 show dielectric constant and specific resistance of muscle tissue, heart muscle, kidney, liver, lung, fat, and blood, as function of frequency. Human autopsy material of normal composition with regard to all the factors which determine the electrical properties at ultrahigh frequencies was used. No time effect could be noticed. The results are, therefore, characteristic for the values as given in the live body. This is to be expected, since the factors which determine the electrical properties at ultrahigh frequencies are not subject to rapid change after death. The biological material was cut in pieces of a size suitable for the test vessel and measured in the majority of cases at room temperature, i.e. at 27° C. ± 1° C. Beef blood was measured with a small amount of heparin added to avoid coagulation. Physiological saline solution (0.9% NaCl) was measured on various occasions and the results are included for comparative purposes. Other measurements with saline solutions of different salt concentration and measurements with distilled water were also carried out.

Also included in the tables are values determined by Herrick, Jelatis, and Lee7 at 1,000 mc. Their measurements were carried out at 37° C. They are corrected to a temperature of 27° C., utilizing the temperature coefficients determined by us to permit comparison. These values agree very well with those obtained by us at 900 mc. In general it can be said that the range of variation is surprisingly small for biological material with high water content. This again may be related to the fact that the factors of importance for the uhf-impedance of biological material (H2O-salt-nonaqueous matter) are not subject to excessive variation. It is noted that liver covers a wider range of values which we may relate to the fact that its glycogen and fat content varies somewhat. Lung tissue is very much affected by its air content, which lowers ε and increases ρ as compared to other tissues. The sample included in the Tables was in a rather deflated condition, and may, therefore, be con

### Table I

**Dielectric Constant of Various Tissues as Function of Frequency at 27° C.**

<table>
<thead>
<tr>
<th>Tissue</th>
<th>ε</th>
<th>Samples</th>
<th>200</th>
<th>300</th>
<th>400</th>
<th>600</th>
<th>700</th>
<th>900</th>
<th>1000 Mc (Herrick)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Muscle</td>
<td>2</td>
<td>56</td>
<td>55-57</td>
<td>54-56</td>
<td>55-56</td>
<td>55-56</td>
<td>55-56</td>
<td>55-56</td>
<td>53-55</td>
</tr>
<tr>
<td>Heart M.</td>
<td>3</td>
<td>59-63</td>
<td>55-62</td>
<td>54-58</td>
<td>54-58</td>
<td>54-58</td>
<td>54-58</td>
<td>54-58</td>
<td>53-58</td>
</tr>
<tr>
<td>Kidney</td>
<td>2</td>
<td>62</td>
<td>57-60</td>
<td>55-57</td>
<td>55-57</td>
<td>55-57</td>
<td>55-57</td>
<td>55-57</td>
<td>55-57</td>
</tr>
<tr>
<td>Lung</td>
<td>1</td>
<td>35</td>
<td>36</td>
<td>36</td>
<td>36</td>
<td>36</td>
<td>36</td>
<td>36</td>
<td>35</td>
</tr>
<tr>
<td>Blood</td>
<td>1</td>
<td>67</td>
<td>63</td>
<td>64</td>
<td>62</td>
<td>62</td>
<td>62</td>
<td>62</td>
<td>63</td>
</tr>
<tr>
<td>0.9% NaCl</td>
<td>2</td>
<td>75</td>
<td>77</td>
<td>74</td>
<td>79</td>
<td>77</td>
<td>77</td>
<td>77</td>
<td>3.2-63</td>
</tr>
<tr>
<td>Fat</td>
<td>3</td>
<td>4.5-7.5</td>
<td></td>
<td></td>
<td>4-7</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

### Table II

**Specific Resistance of Various Tissues as Function of Frequency at 27° C.**

<table>
<thead>
<tr>
<th>Tissue</th>
<th>ρ</th>
<th>Samples</th>
<th>200</th>
<th>300</th>
<th>400</th>
<th>600</th>
<th>700</th>
<th>900</th>
<th>1000 Mc (Herrick)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Muscle</td>
<td>2</td>
<td>110-120</td>
<td>105-110</td>
<td>100-105</td>
<td>94-100</td>
<td>87-93</td>
<td>81-84</td>
<td>79-83</td>
<td></td>
</tr>
<tr>
<td>Heart M.</td>
<td>3</td>
<td>110-130</td>
<td>105-120</td>
<td>100-115</td>
<td>95-115</td>
<td>92-110</td>
<td>83-100</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Liver</td>
<td>4</td>
<td>125-170</td>
<td>120-155</td>
<td>120-150</td>
<td>110-140</td>
<td>100-130</td>
<td>92-120</td>
<td>104-110</td>
<td></td>
</tr>
<tr>
<td>Kidney</td>
<td>2</td>
<td>104</td>
<td>100-102</td>
<td>98</td>
<td>90-94</td>
<td>89-90</td>
<td>81-82</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Lung</td>
<td>1</td>
<td>190</td>
<td>170</td>
<td>163</td>
<td>156</td>
<td>152</td>
<td>137</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Blood</td>
<td>1</td>
<td>96</td>
<td>90</td>
<td>91</td>
<td>92</td>
<td>85</td>
<td>80</td>
<td>70-78</td>
<td></td>
</tr>
<tr>
<td>0.9% NaCl</td>
<td>2</td>
<td>58</td>
<td>59</td>
<td>58</td>
<td>56</td>
<td>54</td>
<td>56</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Fat</td>
<td>3</td>
<td>1500-5000</td>
<td>1300-4000</td>
<td>1300-4000</td>
<td>1300-4000</td>
<td>1300-4000</td>
<td>1300-4000</td>
<td>1300-4000</td>
<td>54</td>
</tr>
</tbody>
</table>
considered as a sample with high $\epsilon$ and low $\rho$ as compared to actual in situ conditions. Another material whose electrical data vary considerably is fat. This is probably because the water content of fatty tissue varies extremely. The values of all tissues with high water content are around 50 (dielectric constant) and 100 ohm cm (specific resistance). Fat, a material with low water content, has a higher resistivity and lower dielectric constant.

Fig. 4 shows in greater detail the frequency dependence of muscle and liver. The curves are shown to demonstrate the typical frequency behavior found in tissue with high water content. It is seen that the dielectric constant is constant at high frequencies and often increases as the frequency decreases below 300 mc. The resistivity curve indicates the superposition of two major frequency variations. One occurs at frequencies far in excess of 300 mc and one occurs predominantly far below 300 mc.

![Graph](image)

Fig. 4—Dielectric constant and specific resistance of muscle and liver tissue as function of frequency.

**Analysis of Results**

The structure of tissues with high water content is determined by cell envelopes which have rather high capacity and resistance and are extremely thin. Inside and surrounding these cell membranes are salt solutions of an ionic strength comparable to physiological saline solution (0.9% NaCl). These solutions contain protein molecules, i.e. macro-molecules of considerable size and high molecular weight. The shape of these proteins may be approximated by ellipsoids of revolution with an axis ratio varying from 1 to 10.

The influence of the cell membranes on the electrical impedance has been investigated in a great number of articles in the past (see for example\textsuperscript{18,19,20}). A somewhat over-simplified model circuit characterizes their influence (Fig. 5). In this circuit the membranes are represented by capacitances, the interior of the cells by a resistance in series with the membrane capacitances, and the exterior fluid by the resistance $R_0$ in parallel with a capacitance $C_\infty$. Dielectric constant and resistance, defined in an equivalent parallel RC-combination follow the laws given in Fig. 5. The subscripts 0 and $\infty$ indicate values of dielectric constant and conductivity $\kappa = 1/\rho$ as determined at extremely low and high frequencies. $T$ is a time constant, essentially identical with the product $RC$ of the circuit, and determines the angular frequency $\omega_0 = 1/T$ where the change with frequency is most pronounced. Actually, it has been shown that either a series of time constants, varying with cell size and shape, or a more complicated impedance for the cell membrane exist. As frequency increases, the capacity of the interior and exterior fluid cannot be neglected. But these are refinements which do not affect the basic phenomena of a decrease of both capacity and resistance with increasing frequency to a constant level. The range of major change has been found to exist in biological material around 10 kc to 10 mc, while near 100 mc, constant values are approached. The change of both electrical data which occurs as the frequency decreases below 300 mc is, therefore, due to the inhomogeneous structure of the tissues (cell membrane capacities).

![Diagram](image)

Fig. 5—Simplified equivalent circuit and equations characterizing frequency dependence of tissue and blood due to cellular structure.

As the frequency increases more and more, another effect appears, which is related to the electrical polarity of water molecules. As changes in field direction become more rapid, inertia eventually makes it impossible for the dipolar molecules to follow the alterations of the electrical field and a consequent decrease in electrical polarization occurs. Here again the relationship (1) applies. However, the parameters have different values.\textsuperscript{21} Both "dispersion" ranges are separated by a more or less

\textsuperscript{18} H. Fricke, "The electric conductivity and capacity of disperse systems," \textit{Physica}, vol. 1, p. 106; 1931.


\textsuperscript{21} $\epsilon_0$ of the "structural dispersion" is identical with $\epsilon_0$ of the polar dispersion. The "characteristic wavelength" (sprungwellenlaenge) of the polar dispersion is near 2 cm while it varies between 100 m and 10,000 m in the case of structural dispersion of tissue with high water content and blood.
pronounced frequency region where the electrical data are relatively frequency independent. This flat level is very obvious in the case of the dielectric constant since it follows from the equations (1) that the dielectric constant begins to decrease at first at frequencies above 1,000 mc.\(^2\) The resistance on the other hand decreases much more rapidly in agreement with the equations for the polar dispersion.\(^2\) An electrolytic solution with a resistivity similar to that of tissue has been calculated for comparative purposes and the result is indicated by the dashed line in Fig. 4, proving that the resistance change of tissues above 300 mc is due to its water content.

A third effect must be discussed, aside from the effect of structural components (cell membranes) and polarity of water. Protein molecules contribute about 20 gm per 100 cc volume in tissue with high water content. The partial volume of protein molecules is about 0.75. The amount of water which is fixed to protein and can be considered insoluble for salts may be assumed in the neighborhood of 0.3 gm per gram protein.\(^9\)\(^,\)\(^23\)\(^,\)\(^24\)\(^,\)\(^25\) The volume which does not participate in electrical conduction is, therefore, in the vicinity of 20 cc per 100 cc material. We can assume that the protein and the bound water establish particles of ellipsoidal shape.\(^2\)\(^6\) Considering that electrical conductivity and dielectric constant of the hydrated protein are much smaller than those of the surrounding saline solution, Fricke's equation\(^2\)\(^6\)\(^,\)\(^9\)

\[
K_t = K_s \frac{1 - \rho}{1 + f \rho}
\]  

(2)

can be applied. Here, \(K\) may be either conductivity or dielectric constant, \(\rho\) is the relative volume occupied by the hydrated protein and the indices \(t, s\) indicate total solution and solvent, \(f\) is a factor which depends on the shape of the protein particles. It is 0.5 for spherical form and increases to a value of 1 for elongated particles. From this, it follows that the presence of protein molecules reduces dielectric constant and conductivity of the surrounding saline solution 27 to 33 per cent. This, combined with the fact that the dielectric constant of saline is 77 at 27\(^\circ\) C. leads to a prediction of 52 to 56 for the dielectric constant. This prediction applies, of course, only to the tissues with high water content in the frequency range where neither structural nor dipolar dispersion affect the data, i.e. above 300 and below 1,000 mc. The values for heart, muscle, and kidney fall in predicted range. The liver values extend down to 44. This may be due to the relatively large amount of glycogen and fat which has not been considered in our argument and which causes a further reduction. Lung is appreciably lower due to its high air content in spite of the fact that it was measured in a state of collapse and consequently deflated. The protein content for blood is lower, about 15 g per 100 cc and the majority of its proteins (hemoglobin) nearly spherical. Hence, a dielectric constant of about 60 is anticipated. This is in agreement with the result and similar investigations carried out by Cook\(^9\) and others.\(^7\) A similar calculation permits determination of the resistance of the fluid surrounding the protein molecules. It is found to be near 70 ohm cm and about 20 per cent higher than the resistance of 0.9 per cent saline solution. This difference is due to the fact that potassium replaces in the intracellular fluid a large part of the sodium ions with different ion mobility.

**Temperature Studies**

It is desirable to obtain data at different temperatures, to permit a transfer of the results to other temperatures. Measurements at different temperature levels were, therefore, conducted. The results obtained apply to fatty tissue samples, kidney, heart muscle, and liver tissue. Fig. 6 shows a typical result obtained with kidney tissue. The temperature coefficient of the resistance varies with the frequency. It is always negative and comparable to that of saline solution. This is in agreement with the analysis which has been advanced above.

\[\text{Fig. 6—Dielectric constant and specific resistance of kidney tissue as function of frequency and temperature.}\]

The temperature coefficient of the capacitance is positive at low frequencies, becomes zero, and finally negative as the frequency increases. The explanation of the data at the high frequencies is based on the fact that the temperature coefficient of the dielectric constant of

\[\text{Fig. 6—Dielectric constant and specific resistance of kidney tissue as function of frequency and temperature.}\]
saline solution is negative and has the same value within the accuracy of our measurements. At lower frequencies a somewhat more complicated situation exists. It has been shown that the temperature coefficient of the capacity of blood at 1 kc and consequently the capacity of the erythrocyte membrane is practically temperature independent.\(^1\) Similar results have been obtained by us with other tissues (unpublished material). The variation in speed with which the biological membranes are charged at different temperatures is determined only by the variation of the extra- and intra-cellular fluid resistance with the temperature. (The time constant \(T = R \times C\) in the circuit Fig. 5 changes proportionally with \(R\) as the temperature varies.) This means that the total dispersion curve due to the structure of the biological material shifts to higher frequencies by a ratio \(f_1/f_2\) which is equal to the ratio \(R_2/R_1\). The \(R\)-ratio may be taken either from the two \(\rho\)-curves in Fig. 6 or directly from tables which give the resistivity of saline solutions as function of temperature. This ratio is about 1.3 for a change from 27\(^\circ\) to 42\(^\circ\) C. It is possible, therefore, to predict from one dispersion curve of \(\epsilon\) others at different temperatures simply by shifting it in frequency as outlined above, and changing their ordinate by a constant value as determined by the temperature coefficient of the dielectric constant of water. The curve in Fig. 6 which is given for a temperature of 42\(^\circ\) C has been determined that way from the 27\(^\circ\) C curve. The curve fits with the experimental values within the accuracy of the determination. This may be considered as another support for the analytical arguments advanced above.

Table 3 summarizes the temperature coefficients as determined in various tissues with high water content.

<table>
<thead>
<tr>
<th></th>
<th>Heart</th>
<th>Kidney</th>
<th>Liver</th>
<th>Average</th>
<th>Saline</th>
<th>(\Delta\rho/\rho)</th>
<th>(\Delta\epsilon/\epsilon)</th>
</tr>
</thead>
<tbody>
<tr>
<td>200 (\mu)</td>
<td>-1.5</td>
<td>-2.0</td>
<td>-1.8</td>
<td>-1.8</td>
<td>-1.7</td>
<td>-1.3 (\times 10^{-4}/^\circ)C</td>
<td>-0.4 (\times 10^{-4}/^\circ)C</td>
</tr>
<tr>
<td>400 (\mu)</td>
<td>-1.3</td>
<td>-2.0</td>
<td>-1.8</td>
<td>-1.7</td>
<td>-1.6</td>
<td></td>
<td></td>
</tr>
<tr>
<td>900 (\mu)</td>
<td>-1.0</td>
<td>-1.3</td>
<td>-1.4</td>
<td>-1.2</td>
<td>-1.3</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

The values are compared with the temperature coefficient of saline solution and fit well into the picture outlined above. The positive temperature coefficient of the dielectric constant is never larger than 0.2. The temperature coefficient of the resistivity is always negative and in the neighborhood of 1.7 per cent per degree C. for frequencies lower than 400 mc. At higher frequencies it decreases in agreement with the theoretical prediction. This is a result of the fact that the temperature coefficient of saline solution decreases when polar losses add substantally to the ionic conductivity. The values given for saline solution are theoretically calculated from the dispersion equation assuming a low frequency conductivity of about 80 ohm cm, which is near the value indicated for cellular fluid as outlined above.

The temperature coefficient of fat cannot be explained on the basis of the model proposed for the tissue with high water content. The coefficient for the capacitance decreases slightly as the frequency increases and varies from 1.3 per cent at 150 mc to 1.1 per cent at 900 mc. The temperature coefficient of the resistivity is rather large and negative. Its value is \(-4.9\) per cent at 150 mc and \(-4.2\) per cent at 900 mc.

**Conclusions**

1. Electrical data are presented for a number of tissues over a frequency range from 200 to 900 mc.
2. The temperature coefficient of dielectric constant and resistivity is determined for various tissues over the frequency range from 200 to 900 mc.
3. An analysis is given which explains the frequency behavior and the temperature coefficients of both dielectric constant and resistivity of water content. It is presented in terms of parameters which characterize the inhomogeneous structure of the material, the polar properties of water molecules, and the protein content.
4. The results show the similarity of the dielectric properties of all tissues with high water content and whole blood. Previously formulated opinions concerning the relative heating of fat muscle layers with radiation diathermy are, therefore, justified.

**Acknowledgment**

We are indebted to Dr. D. L. Drabkin and Dr. J. B. Marsh, of the Department of Physiological Chemistry, Medical School, University of Pennsylvania, who have carried out protein determinations of various tissue samples and have been extremely helpful in discussions concerning proteins in tissue and blood.

Furthermore, we express our appreciation to Dr. W. F. Sheldon of the Department of Pathology, School of Medicine, University of Pennsylvania, and Dr. W. Beckfield, Department of Pathology, Philadelphia General Hospital, for their preparation of the tissue samples.

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\(^1\) H. P. Schwan, "Die Temperaturabhängigkeit der Dielektrizitätskonstante von Blut bei Niederfrequenz," *ZS. Naturforschung*, vol. 3b, p. 361; 1948.