Introduction

In the whimsically titled letter “Holmes & Watson reply to Sherlock” (Holmes et al., 1998) my colleagues and I made a serious critique of Halber et al. (1997), a paper evaluating thresholding methods for PET activation data. The paper directly compared a nonparametric permutation method (named “Sherlock”), which provided inferences fully corrected for multiple testing, to uncorrected $P_{0.05}$ inference, finding that the latter method was to be preferred for its power. In response to our letter, the paper’s authors defended the uncorrected approach as the (then) default setting in the SPM1 software and claimed that it had been used in “approximately 1200 publications”.

Over a decade later, and one “Voodoo correlations” (Vul et al., 2009) imbroglio and post-mortem ichthyological fMRI study (Bennett et al., 2011) later, it seems everyone agrees that (a) correcting inferences for the search over the brain is essential and (b) such corrections are not consistently utilized in fMRI. Hopefully some historical perspective can strengthen the discipline’s resolve to uphold good statistical practice.

What follows is a highly selective review of the literature on the multiple testing problem in fMRI and its antecedents (PET and M/EEG). I have tried to capture the major landmark publications, and while this selection is inevitably quirky and personal, I hope it will provide a useful perspective in this important aspect of fMRI data analysis. See Holmes (1994) and Petersson et al. (1999) for more careful and detailed reviews of early work in this area.

The problem

Whether studying brain structure or brain function, using MRI, PET or M/EEG modalities, the end result of an experiment is typically a set of statistic values (e.g. $T$ or $F$ values) that comprises an image. This “image” may be a 2D surface, a 3D volume, or even a 4D movie of statistics over time. Call $T_i$ the statistic image, with $T_i$ the value at voxel $i$. Before even mentioning “multiple testing” we must define the objects under inference. There are in fact a variety of...
Assessing statistical images: voxels, clusters and peaks

Voxel-wise inference uses a threshold \( u \) and classifies voxel \( i \) as “active” if \( T_i \geq u \); inference is made on each voxel individually. Cluster-wise inference uses a cluster-forming threshold \( u_c \) to define blocks, i.e. contiguous suprathreshold regions. If \( S \) is the size of a cluster, cluster inference consists of retaining all clusters with \( S \geq k \) for some cluster size threshold \( k \). For voxel-wise inference, when \( T_i \geq u \) we can make a statement about the signal at voxel \( i \). For cluster-wise inference, when \( S \geq k \), we are making a statement about a “random set”, the collection of voxels in the cluster. With replication of our experiment, voxel \( i \) still means voxel \( i \), but a cluster will comprise different voxels if it exists at all.

So what exactly is the interpretation of a significant cluster? I usually answer “that one or more voxels within that cluster have evidence against the null” (Poldrack et al., 2011); that is, the test can localize the effect to somewhere within the cluster. This lack of precise spatial specificity is a shortcoming, but voxel-wise inference has its critics too. Friston and colleagues have argued against voxel-wise inference (Chumbley and Friston, 2009; Chumbley et al., 2010), saying that for smoothed data a voxel is ill-defined, and only topological quantities are interpretable, like peaks (local maxima) or clusters. As peaks, like clusters, are randomly located, and as voxels have reasonably compact point spread functions in practice, I counter that voxel-wise inference remains a useful approach.

Peak-wise inference is based only on local maximum above a given screening threshold \( u_p \). Peak-wise is not the same as voxel-wise inference\(^2\) and the inference will depend on the chosen \( u_p \) threshold. Finally, set-wise methods, based on just the count of clusters, and other omnibus measures can be defined, but they do not have any localizing power.

Statistics, \( P \)s and corrected \( P \)s

Once a method for assessing the statistic image is chosen, a test statistic needs to be defined. For voxel-wise or peak-wise inference, the statistic value is obvious (just the value of \( T \) at the voxel, or the peak), and for cluster-wise inference this is naturally the number of voxels in the clusters (though there are other ways; see below).

Based on a test statistic an uncorrected \( P \)-value \( P \) can be defined. For example, suppose we are performing voxel-wise inference; for a randomly (or \( a \) priori) selected voxel \( i \), the \( P \)-value \( P_i \) is the chance of observing a test statistic \( T_i \) as or more extreme, assuming that the null hypothesis is true. For voxel-wise statistic \( T_i \) this is a trivial computation, even possible with a table in the back of a textbook. For a peak value or cluster size, however, no standard results are available. Before reviewing the tools imagers used to find uncorrected \( P \)-values, let me first introduce an even greater challenge, the multiple testing problem.

Whether voxel-wise or cluster-wise, there is a huge multiplicity. Searching over 100,000 voxels in the brain we expect to find 5000 \( P \)’s smaller than 0.05 even in the null scenario of no activation. Likewise, searching over 100 clusters will on average produce 5 uncorrected cluster \( P \)-values less than 0.05. To account for the multiplicity, we have to define a measure of error when searching the brain. The standard measure is the False Discovery Error Rate (FWE), the chance of one or more false positives (Nichols and Hayasaka, 2003). FWE is the quantity controlled by the well-known Bonferroni procedure, and while it is a sensible measure of false positives, many find it lacks power.\(^4\)

The False Discovery Rate (FDR) is a more lenient measure of false positive risk, defined as the expected proportion of false positives among detections (Benjamini and Hochberg, 1995).\(^5\) My colleagues and I (Genovese et al., 2002) introduced FDR to functional neuroimaging, and I see its wide embrace as a sign of how hungry users were for calibrated multiple testing procedures that are more powerful than FWE.

Another less-used alternative to FWE is the expected number of false positives (Bullmore et al., 1996). This measure is used in the CamBA software\(^6\) to control the expected number of false positive clusters at just below 1.0.

For either FWE or FDR, you can define corrected \( P \)-values for a particular \( T_i \) (or peak value or cluster size); the smallest FWE (or FDR) \( \alpha \) level that will just reject the null hypothesis for \( T_i \).

And what about poor old uncorrected \( P \leq 0.001 \), with perhaps some cluster threshold like \( S > 10 \) voxels? In principal, the false positive risk of any fixed heuristic could be validated with a sufficient amount of real null data, and then the heuristic could safely be applied to data with the very same characteristics. But if any aspect of the data changes — voxels size smoothing FWHM, number of slices or their orientation orientation — then the false positive risk will vary in some undetermined way. Hence, the best practice dictates the use of multiple-testing corrected inferences that have the same interpretation for all data.

Finally, it should be noted that an entirely different approach to inference is taken with the Bayesian paradigm (see Woolrich, 2011 for a review). Instead of pretending the null hypothesis is true in order to compute \( P \)-values, the Bayesian approach focuses on estimating and characterizing the uncertainty of parameters in the non-null state. While some have argued that a Bayesian approach avoids the multiple testing problem altogether (Friston et al., 2002), it remains a problem if you consider a Bayesian decision theoretic framework with a loss function measuring false positive risk over the image (see, e.g., Muller et al., 2006).

A preview of solutions

The crux of methodological research in neuroimaging inference has been how to find thresholds on test statistics that control a specified error rate. Before a historical tour of this research, it’s helpful to lay out the three broad types of approaches that have been used.

The best known (if least understood) approach is Random Field Theory (RFT). In rough terms, RFT uses the smoothness of the image noise to predict the behavior of extreme values of voxel-, peak- and cluster-wise statistics. The underlying theory is elegant and has connections to topology but requires that, in addition to the usual Gaussian assumption, the image data behave like a continuous random process (i.e. are smooth).

The other frequently used approach is Monte Carlo (MC). By estimating basic features of the data under the null hypothesis, like image smoothness, MC repeatedly simulates null replicates of the data. The observed test statistics (peak, cluster, whatever) can then be compared

\(^{2}\) Despite the obviousness of this comment, I know of no formal proof that cluster inference has such strong control of Family wise error. I will make ample use of footnotes to comment on such minutiae.

\(^{3}\) Jumping ahead, FWE-corrected peak-wise \( P \)-values equal FWE-corrected voxel-wise \( P \)-values at the peaks. This is because FWE is determined by the distribution of the maximal statistic, and the maximum voxel-wise is the maximum peak-wise.

\(^{4}\) People often say “FWE is conservative”, but that’s like saying a meter is too short.

\(^{5}\) The work was circulating in statistics circles well before 1995; see Benjamini (2010) for some history.

\(^{6}\) http://www-bnm.psychiatry.cam.ac.uk/software/.

\(^{7}\) The reasoning is as follows: Because posterior inferences are a function of the observed data, which is fixed, there is no random outcome from which a FWE probability can be computed. Put another way, the posterior probability for an inference computed for one voxel is the final statement based on that dataset and no “correction” is applicable.
to the simulated null distributions, creating P-values. Just like RFT, Gaussianity has to be assumed and the smoothness has to be estimated, but MC doesn’t depend on the accuracy of RFT approximations. Finally, there is the permutation test. Using the data itself, empirical null distributions are created by permuting (or otherwise altering) the data under the null hypothesis. This approach has the weakest assumptions and is growing in use, but has limitations, in particular in dealing with time series autocorrelation and general experimental designs.

A tour of solutions

Early days

Many “fMRI statistical methods” are in fact generic procedures developed first for PET. Hence we start with seminal work by Fox and Mintun (1989), who showed that non-quantitative $H_2^3O$ PET could be used to map brain function. As part of that paper they proposed “Change Distribution Analysis” to determine if there were any effects in the image. They used the distribution of all local extrema, that is, the value of local maxima for $T_i>0$ and all local minima for $T_i<0$ (no screening threshold $t_p$). Defining global skew and kurtosis statistics on the distribution of peak values, and using conventional standard errors they produced an omnibus test for activation in the brain.

Random Field Theory

Change Distribution Analysis lacked any localization power, and of course there was a need for methods that would assign significance locally, to each voxel, while still controlling FWE. Friston et al. (1991) solved this problem using general theory of Gaussian processes, working in 2D and assuming equal smoothness in X and Y directions. Shortly afterwards, Worsley et al. (1992) produced a more general 3D solution that would define a class of methods: Random Field Theory. By drawing a connection between the voxel-wise FWE and the expected Euler characteristic, Worsley created inferences that accounted for both the volume and smoothness of the search region. He created the notion of a Resolution Element, or RESEL, a virtual voxel with dimensions equal to $\text{FWHM}_x \times \text{FWHM}_y \times \text{FWHM}_z$.

In the PET data Worsley and colleagues were using, there seemed to be no evidence for spatially varying variance. Hence the initial 1992 work assumed the variance estimate could be pooled over the entire brain, producing a $Z$ statistic image. Others groups found PET data to have spatially varying standard-deviation, and, in particular, Friston et al. (1991) used a voxel-wise variance estimate; at the time there were no results for the resulting $T$ image, so the $T$ was Gaussianized to create $Z$ results. Worsley and coauthors soon generalized his results to account for voxel-wise variance estimation, for $T$, and $F$ images (Worsley et al., 1993; Worsley, 1994), though these results didn’t make their way into Friston’s SPM until “SPM99” and FSL still uses the Gaussianization. See Table 1 for a tale how Worsley and Friston came to be collaborators after this potentially fractious beginning.

Monte Carlo

Voxel-wise thresholding couldn’t detect low-intensity, spatially extended effects. In lieu of theoretical results, a Monte Carlo simulation approach was proposed first for PET (Poline and Mazoyer, 1993; Roland et al., 1993) and then for fMRI (Forman et al., 1995). Using an estimate of the smoothness of the data, simulated statistic images under the null hypothesis generate an empirical estimate of the maximum cluster size, from which cluster size statistics can be converted to FWE-corrected $P$-values. This approach is still used today in the AFNI software’s alphavis. In their first joint work, Worsley and Friston (and colleagues) used Random Field Theory to produce closed-form FWE $P$-values for cluster size statistics (Friston et al., 1993).

An entire separate review paper is needed to track all the RFT work produced, but a few highlights include: A unified result for $Z$, $T$, $x^2$ and $F$ images (Worsley et al., 1996); a solution for the conservativeness found at low smoothness (Worsley and Taylor, 2005); and a unified multivariate result from which all other results are special cases (Worsley et al., 2004). These methods and more are implemented in surfstat, a program Worsley was actively developing until his death in 2009.

Whether Monte Carlo or RFT, the estimation of smoothness is crucial. Poline et al. (1995) found that if smoothness was estimated from but a single image (as was done in SPM99), RFT $P$-values should have confidence intervals of about $\pm 40\%$. This uncertainty affects

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Table 1

| KJF on KJW. Keith Worsley and Karl Friston authored foundational papers in the 1990s on inference for neuroimaging. We lost Keith suddenly in 2009, so I asked Karl to comment on how it was that a psychiatrist and a statistician came to be friends and collaborators. |

*Keith and I first met in 1990 at a workshop at Harvard Medical School. I was chaperoned by Richard Frackowiak and Keith by Alan Evans. Alan had famously recruited Keith after finding him collecting maple leaves on the campus of McGill University—in the fond hope of finding something interesting to study! Keith had seen the potential of random field theory and had been sent a final draft of my 1991 paper (Friston et al., 1991). I remember him being very excited by the prospect of applying random field theory to neuroimaging data. He was also bemused and intrigued by the divergence of the general theory of stochastic processes and level sets (my 1991 paper) and random field theory proper (his paper, Worsley et al. 1992). I also recall him being exercised by a mild discrepancy between the two formulations; the discrepancy boiled down to a square root two factor that he could not resolve, and remains unresolved two decades later. We became friends over the ensuing months, or more exactly ‘pen-pals’. Getting emails from Keith was a bit like playing Russian roulette. Most of the time they were insightful, reassuring and helpful but—occasionally—he would start with ‘I think there’s a small problem…’. What he meant was that there was a substantial conceptual or technical problem that would take at least six months hard work to resolve. The substantial exchanges between us often weren’t reflected in publications or the rhetoric we each developed, perhaps to underscore the distinct contributions of our respective groups. It is worth remembering that we were separated not just by the Atlantic but by some esthetic and pragmatic differences. For example, we always assumed that error variance was regionally specific, but Keith never liked this, because it destroyed some of the simple beauty of implementing the theory. On the other hand, Keith loved the most advanced graphics software that he could find, whereas we stuck religiously to Matlab despite its very limited graphics support (at that time). Years later, the intellectual collaboration rested on shared students and fellows, like Jean-Baptiste Poline and Stefan Kiebel. Much of that work is embodied in SPM and has remained the mainstay of topological inference using random field theory to date.*
Monte Carlo P-values to the same or greater degree. Standard practice now is to estimate smoothness from standardized residual images (Kiebel et al., 1999), but there remain two different approaches. Forman et al. (1995) estimated the smoothness based on a discretized Gaussian kernel, where the estimator of Kiebel et al. (1999) is based on partial differences approximating a continuous random field’s derivatives. While the latter makes no assumption about the shape of the autocorrelation function—except the existence of 2 derivatives at the origin—it has greater bias at low smoothness.15

Cluster-wise inference captures the spatial nature of the signals, and suffers from less multiplicity than voxel-wise inference. However it is not always more sensitive, and Friston et al. (1996) showed that the power of cluster inference depends on the spatial scale of the signal relative to the noise smoothness: Focal, intense signals will be better detected by voxel-wise inference. Thus there is a natural temptation to compute both cluster-wise and voxel-wise results and take the better of the two. This of course forms a new multiple testing problem, which will yield more false positives.16 To address this, Poline et al. (1997) proposed a RFT-based joint cluster size, cluster peak-height test.

Permutation

Inspired by Blair and Karniski’s EEG permutation work (1994), Holmes et al. (1996) proposed a permutation test for PET that controlled FWE with few assumptions. Based on that work, Holmes and I created the SnPM17 software, which we thought would quickly become irrelevant as fMRI came to dominate neuroimaging. The problem was that fMRI times series’ autocorrelation violates a basic assumption needed by permutation, exchangeability. Others had tackled this problem, by decorrelating the fMRI data using the fit of a parametric autocorrelation model (Bullmore et al., 1996; Locascio et al., 1997), however we found this mix of parametric and nonparametric modeling unsatisfactory.18 However fMRI analysis quickly came to focus on group analysis using a summary statistic approach (Holmes and Friston, 1999; Mumford and Nichols, 2009), meaning our PET 1-scan-per-subject permutation methods remained relevant.

In 2001 I was surprised by a OHBM conference poster that showed RFT voxel-wise thresholds were wildly conservative for small n (Stoeckl et al., 2001). Despite their widespread use, RFT methods had actually never been evaluated for use with small sample sizes.19 Using PVW20 consisting of Monte Carlo simulations and real data comparisons with permutation, we replicated the finding of conservative voxel-wise RFT thresholds (Nichols and Hayasaka, 2003; Nichols and Holmes, 2001) and also reported on instability of cluster-wise results (Hayasaka and Nichols, 2003; Hayasaka et al., 2004).21 Despite these results on the power gains of voxel-wise FWE permutation inference over RFT, SnPM did not become an integrative SPM tool.22 In FSL, however, the “randomise” software23 has become a central tool for all voxel-based anatomical analyses. Aside from overcoming any RFT conservativeness, permutation inference works in nonstandard settings like Tract-Based Spatial Statistics (Smith et al., 2006) where tracks are highly irregular and vary in topology from 1-D to 2-D. Permutation also allows consideration of new test statistics, where no parametric result is available. Examples include: The smoothed variance T-test (Holmes et al., 1996), cluster-mass (Bullmore et al., 1999), different peak-cluster combining tests (Hayasaka and Nichols, 2004), and a completely new cluster-inspired method, Threshold-Free Cluster Enhancement (Smith and Nichols, 2009). Permutation even feeds-back into RFT research: We developed a RFT cluster-mass test (Zhang et al., 2009) only after extensive experience with permutation showed that it outperformed alternate peak-cluster combining methods (Hayasaka and Nichols, 2004).

Despite my personal enthusiasm for permutation-based inference, it must be acknowledged that when RFT inference procedures work, they deliver similar answers at a fraction of the computational effort of permutation. Indeed, considering that permutation would be the only approach if RFT had never had been developed, RFT has surely saved 1000’s of years of computation time.

The future

Looking ahead, there is renewed enthusiasm for resampling-based test as GPU’s make order-of magnitude speed-ups (Eklund et al., 2011), and in particular which make local multivariate methods attractive (Eklund et al., 2011; Nandy and Cordes, 2007).

Predictive analyses and “brain reading” distill inference to a single accuracy number (Haynes and Rees, 2006) and seem to be a step away from “brain mapping”. But in practice investigators wish to determine which brain regions are responsible for the predictive power, and thus we return to a spatial mapping exercise (Kriegeskorte et al., 2006).

And perhaps the most promising direction is the application of explicit spatial models to brain image data, for both original fMRI data (Keller et al., 2008; Xu et al., 2009; Weeda et al., 2009; Thirion et al., 2010; Kim et al., 2010; Gershman et al., 2011) and meta-analysis data (Neumann et al., 2008; Kang et al., 2011). These methods can provide spatial confidence intervals on effects of interest and more flexible and interpretable model fits.

Finally, I apologize to the authors of scores of papers on fMRI inference that I have not cited. Sometime in the next 20 years I hope I can make a more comprehensive review.

References


15 SPM and AFNI use the Kiebel approach, though SPM only uses up to 64 images by default; FSL uses a version of the Forman approach on the standardized residuals (Flimley et al., 2000; Jenkinson, 2000).
16 The SPM software encourages profligate exploration of results, showing all possible types of inferences, while the FSL software only provides users one of voxel-wise or cluster-wise inferences.
17 http://go.warwick.ac.uk/tenichols/software/snpm.
18 More flexible wavelet decorrelation can whiten better (Bullmore et al., 2001), but can have problems with simple block designs (Friman and Westin, 2005). Also note that a randomized experimental design justifies a randomization test with any data (Raz et al., 2003), though this has limited application.
19 Worsley et al. (1992) used Gaussian simulations to verify his Z results, but these only applied to the case of pooling variance over the whole brain or very large n.
20 Probability Validation Work.
21 See also later work showing problems with cluster-wise RFT even with n in the 100’s (Silver et al., 2010).
22 In part it remained difficult to use.
23 Initially an exercise for Tim Behrens to teach Steve Smith C++; I gave instructions from the sidelines.