Soft Ferrites

Properties and Applications

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1.2.2. Magnetization

Electrons spin about an axis and, by virtue of this spin and their electrostatic charge, exhibit a magnetic moment. Normally, in an ion with an even number of electrons, the spins or moments cancel, and when the number of electrons is odd there will be one uncompensated spin. For the transition metals the number of uncompensated spins is larger, e.g. the trivalent Fe ion has a moment equivalent to five uncompensated spins.

When the atoms of these transition metals are combined in metallic crystals, as they are, for example, in iron, the atomic moments are spontaneously held in parallel alignment over regions within each crystallite. The net number of uncompensated spins will be less than for the isolated ion due to the band character of the electron energies in a metal. The regions in which alignment occurs are called domains and may extend over many thousands of unit cells. The spin orientation is along a direction of minimum energy, i.e. external energy is required to deflect the magnetization from this direction and if the external constraint is removed the magnetization will return to a preferred direction. This directional or anistropic behaviour may arise from a number of factors. Crystal anisotropy is inherent in the lattice structure; the magnetization always preferring the cube diagonal or cube edge. Mechanical strain can cause anisotropy and the shape of the grain boundary will nearly always produce anisotropy. The result is that the magnetization is held to a certain direction, or to one of a number of directions, as if by a spring. The greater the anisotropy, the stiffer the spring and the more difficult it is to deflect the magnetization by an external magnetic field, i.e. the lower the permeability (see Chapter 2 for definitions of permeability, etc.)

The parallel spin alignment implies that the material within the domain is magnetically saturated. The magnetization is defined as the magnetic moment per unit volume and is therefore proportional to the density of magnetic ions and to their magnetic moments. This magnetism arising from parallel alignment is called ferromagnetism.

In a ferrite the metal ions are separated by oxygen ions. As a result of this the ions in the *A* sub-lattice (tetrahedral sites) are orientated antiparallel to those in the *B* sub-lattice (octahedral sites). If these sub-lattices were identical the net magnetization would be zero in spite of the alignment and the ferrite would be classified as anti-ferromagnetic. In the majority of practical ferrites the two sub-lattices are different in number and in the type of ions so that there is a resultant magnetization. Such materials are classified as ferrimagnetic. For example, in the foregoing section it was stated that in the general spinel molecule $MeFe_2O_4$ one metal ion occupies an *A* site while two occupy *B* sites; thus in the case of $MnFe_2O_4$ where both metal ions have 5 uncompensated spins the net magnetization is 5 spins per molecule. This compares with a net moment of 2·2 spins per *atom* in the case of metallic iron. For this reason a ferrite has a much lower saturation magnetization ($\mu_o M_{sat} \approx 0{\cdot}5$ Wb.m^{-2}) than metallic iron (about 2·0 Wb.m^{-2}). However, in spite of the partial cancellation of the spin moments, ferrites possess sufficient saturation magnetization to make them useful in a wide range of applications.

The crystallite is normally divided into a number of domains of various spin orientations, e.g. opposite (180°) and orthogonal (90°), so that the crystallite has very little external field arising from the internal magnetization, i.e.

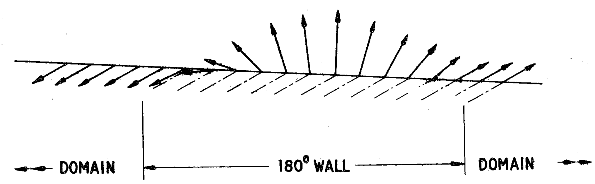
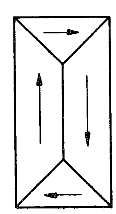


Fig. 1.2. Transition of spin direction at a 180° Bloch wall (domain boundary)

the demagnetizing fields are small. The domain boundaries (Bloch walls) consist of regions many unit cells in thickness in which there is a gradual transition of spin orientation, see Fig. 1.2. This transition must act against the anisotropic forces and the forces which tend to hold the spins in alignment and therefore involves storage of energy. The number and arrangement of domains in a crystallite is such that the sum of the energies, mainly the wall energy and the demagnetizing field energy, is a minimum. Fig. 1.3 shows an idealized arrangement of domains. If an external field is applied, the domain walls

Fig. 1.3. Idealized domain pattern



experience a pressure which tends to make those domains having a component of magnetization in the direction of the field grow at the expense of the unfavourably orientated domains.

In practice it is energetically favourable for domain walls to pass through certain imperfections such as voids, stressed regions, non-magnetic inclusions, etc. Fig. 1.4 is a simplified representation of the situation. In the absence of an applied field the walls are straight and might occupy the positions shown in (*a*). The dots represent imperfections. If a small field is applied in the direction shown (*b*) the walls remain pinned by the imperfections but bulge

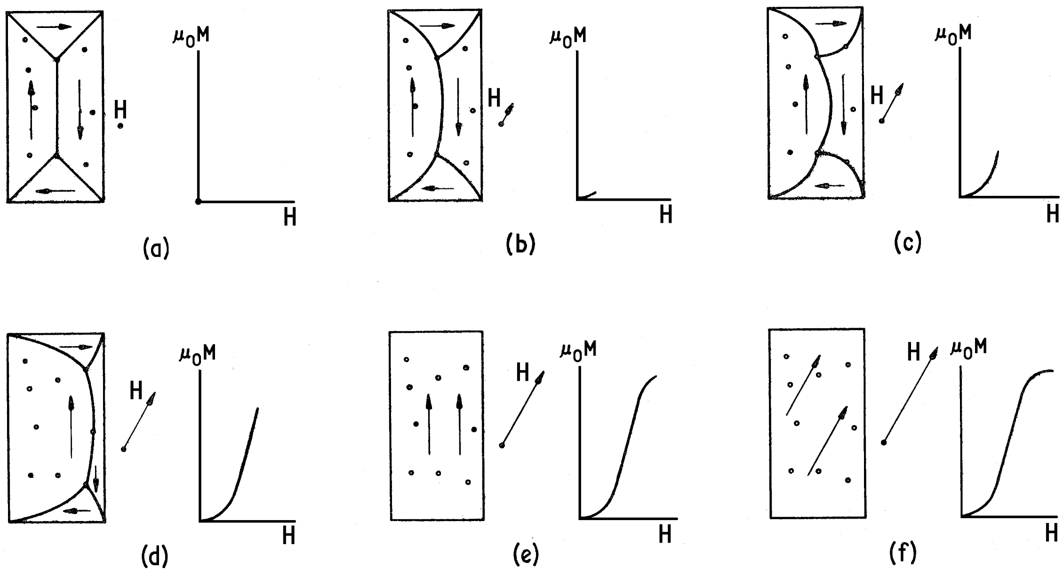


Fig. 1.4. A simplified representation of the part played by domain boundaries in the process of magnetization

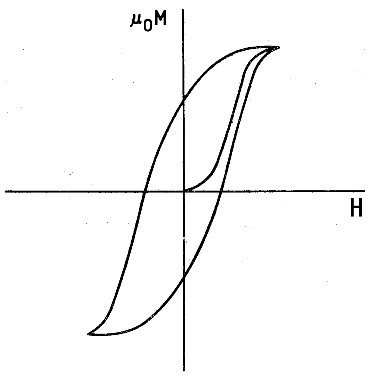


Fig. 1.5. Hysteresis loop

as would a membrane under pressure. These movements are reversible. The change in magnetization is restricted by the stiffness of the walls. Under these circumstances the lower part of the magnetization curve is traced. As the field increases, the pressure on the walls overcomes the pinning effect and the walls move by a series of jumps (*c*, *d*, *e*). These movements are irreversible, i.e. if a certain field change is required to produce a jump, the reversal of that change, i.e. the restoration of the field, will not in general cause the wall to jump back. During this part of the process the magnetization curve rises steeply. Finally, when all the domains have been swept away, further increases in field strength cause the magnetization vector to rotate reversibly towards the external field direction until complete alignment is approached (*f*) No further increase in magnetization is then possible and the material is said to be saturated. Normally in a polycrystalline material there is a wide distribution of grain sizes, domain sizes, orientations, etc., and irreversible and reversible processes merge together. However the above illustration represents the main stages in the magnetization process.

If the magnetic field, having reached the maximum value corresponding to Fig. 1.4(*f*), is made to alternate cyclically about zero at the same maximum amplitude, the initial magnetization curve will not be retraced. Due to the irreversible domain wall movements the magnetization will always lag behind the field and an open loop will be traced. This phenomenon is known as magnetic hysteresis and the loop is called a hysteresis loop, see Fig. 1.5.

The ease with which the magnetization may be changed by a given magnetic field depends on the anisotropy, i.e. magnetic stiffness, whether the change is due to reversible or irreversible wall movements or rotations. A low anisotopy leads to a large induced magnetization for a given magnetic field and therefore to a large value of susceptibility and permeability (see Chapter 2).

1.3. MANUFACTURE

1.3.1. Manufacturing processes

The processes used in ferrite manufacture on an industrial scale are similar to those used in the manufacture of other ceramics. The description of these processes given in this chapter is intended mainly for the information of the user, so that the possibilities and limitations of manufacture may be taken into account when a particular ferrite core design or application is being considered.

Chapter 2

The Expression of Electrical and Magnetic Properties

2.1. MAGNETIZATION

The magnetic field strength, H, inside a very long uniform solenoid having N_1 turns per axial length l and carrying I amperes is given by

$$H = \frac{N_1 I}{l} \qquad \text{A.m}^{-1} \tag{2.1}*$$

Its direction is parallel to the axis of the solenoid and it is uniform across the internal cross section.

The associated flux density, B, is given by

$$B = \mu_o H \qquad \text{tesla† (T), i.e. Wb.m}^{-2} \tag{2.2}$$

where μ_o is the magnetic constant or the permeability of free space. It has the numerical value $4\pi \times 10^{-7}$ and has the dimensions henries/metre or $[LMT^{-2}I^{-2}]$. Thus in the SI units, flux density is dimensionally different from field strength.

If the solenoid is now filled with a magnetic material, the applied magnetic field will act upon the magnetic moments of the ions composing the material. This process has been described qualitatively in Section 1.2.2. The ions, by virtue of the spinning electrons, behave as microscopic current loops each having a magnetic moment. These moments may, in general, be considered to be aligned parallel to each other over small regions, or domains, within the material. In the demagnetized state the domains are distributed so that the vector sum of the magnetization of the domains is zero. Under the influence of an applied field the ion moments are re-orientated, either by the growth and contraction of the various domains or by the rotation of the magnetization within them, so that the ionic moments effectively augment the

*Many of the quantities used in this section are vector quantities and therefore the equations involving them are vector equations. However, in the present limited treatment, the relative directions of the vectors are implied in the text so no special symbols will be used to denote vector quantities or vector operations. For a more general treatment the reader is referred to textbooks on electromagnetic theory.

†The unit of flux density in the SI units has been named the 'tesla' and has the symbol T, (see IEC Publication No. 27). It has the units Wb.m^{-2} so there is no change in its value:

$1\ \text{T} = 1\ \text{Wb.m}^{-2} = 10^4\ \text{Gs}$
$1\ \text{mT} = 10\ \text{Gs}$

(2.1) $H = \dfrac{4\pi N_1 I}{10 l}$ Oe

(1 Oe $\approx$ 80 A.m^{-1})

(2.2) $B = H$ Gs

In the CGS system of units H and B have the same dimensions and therefore the oersted and the gauss are strictly the same units.

applied field. This increase in magnetic field is called the magnetization, M, and it is expressed in $A.m^{-1}$. It is the vector sum of the magnetic area moments* of all the microscopic currents in a given volume of material, divided by that volume. The internal magnetic field, H_i, becomes

$$H_i = \frac{N_1 I}{l} + M \qquad A.m^{-1} \tag{2.3}$$

and the flux density becomes:

$$B = \mu_o H_i = \mu_o(H+M) \qquad T, (Wb.m^{-2}) \tag{2.4}$$

$$\text{or} \quad B = \mu_o H + J \qquad T, (Wb.m^{-2}) \tag{2.5}$$

where J is the magnetic polarization in teslas; it is sometimes referred to as intrinsic flux density

$$J = \mu_o M \qquad T \tag{2.6}$$

Thus M is the increase in the field strength due to the magnetic material and J is the corresponding increase in flux density. The ratio of the magnetization and the applied field strength is called the susceptibility, κ; it is dimensionless.

From Eqn 2.4

$$\frac{B}{H} = \mu_o \left(1+\frac{M}{H}\right) = \mu_o(1+\kappa) \tag{2.7}$$

This quotient of flux density and applied field strength is called the absolute permeability and is sometimes denoted by μ. However it is more usual to show it as the product of the magnetic constant and a dimensionless constant called the relative permeability, μ_r. In the chapters that follow, the relative permeability is such a widely used parameter and is given such a variety of qualifying subscripts that it is convenient to drop the adjective 'relative'. Thus permeability will refer to the dimensionless ratio and in equations it will normally be associated with the magnetic constant, μ_o. The absolute permeability, as such, will not be used.

*Magnetic area moment, m, is the product of a current and the area of the loop in which it flows, the direction is normal to the plane of the loop and when viewed in this direction the current has clockwise rotation.

$$\frac{B}{H} = \mu_o \mu \tag{2.8}$$

from which it follows that

$$\kappa = \mu - 1 \tag{2.9}$$

The applied field strength may be determined by measuring the current and using Eqn 2.1. The measurement of flux density depends on the law of induction, i.e.

$$e = -d\phi/dt \qquad V \tag{2.10}$$

where ϕ is the magnetic flux i.e. the area integral of the flux density; it is expressed in Wb.

In the ideal solenoid $\phi = BA$ where A is the cross-sectional area of the magnetic material. If N_2 turns are wound tightly round the magnetic material the e.m.f. induced will be

$$e = -N_2 A \, dB/dt \qquad V \tag{2.11}$$

By integration, the average e.m.f. during a change of flux density, ΔB, is:

$$\bar{E} = -N_2 A \, \Delta B \qquad V \tag{2.12}$$

The negative sign indicates that the e.m.f. is in such a direction that it would produce current opposing the change of flux. If the flux density is sinusoidal, e.g. if $B = \hat{B} \sin \omega t$, then from Eqn 2.11, dropping the sign:

$$e = N_2 A \hat{B} \omega \cos \omega t = \hat{E} \cos \omega t$$

$$\therefore \hat{E} = \omega \hat{B} A N_2 \qquad V$$

$$\text{or } E = \frac{\omega \hat{B} A N_2}{\sqrt{2}} \qquad V \tag{2.13}$$

If the current in the ideal solenoid is increased from zero, the field strength increases and the magnetization will increase non-linearly by the processes illustrated in Fig. 1.4. It is more usual to consider the dependence of the flux density on field strength. Such a B–H curve is shown in Fig. 2.1. Starting with the magnetic material in an unmagnetized or neutralized state the B–H curve will follow the path ***oba***. If, on reaching the point ***a***, the field strength is decreased, the B–H curve will follow the

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(2.3) $H_i = \frac{4\pi N_1 I}{10 l} + 4\pi M \qquad Oe$

(2.4), (2.5) $B = H_i = H + 4\pi M \qquad Gs$

(2.7) $\frac{B}{H} = 1 + 4\pi \frac{M}{H} = 1 + 4\pi\kappa$

(2.8) $\frac{B}{H} = \mu$

(2.9) $4\pi\kappa = \mu - 1$

(2.10) $e = -\frac{d\phi}{dt} \times 10^{-8} \qquad V$

(2.11) $e = -N_2 A \frac{dB}{dt} \times 10^{-8} \qquad V$

(2.12) $\bar{E} = -N_2 A \Delta B \times 10^{-8} \qquad V$

(2.13) $E = \frac{\omega \hat{B} A N_2}{\sqrt{2}} \times 10^{-8} \qquad V$

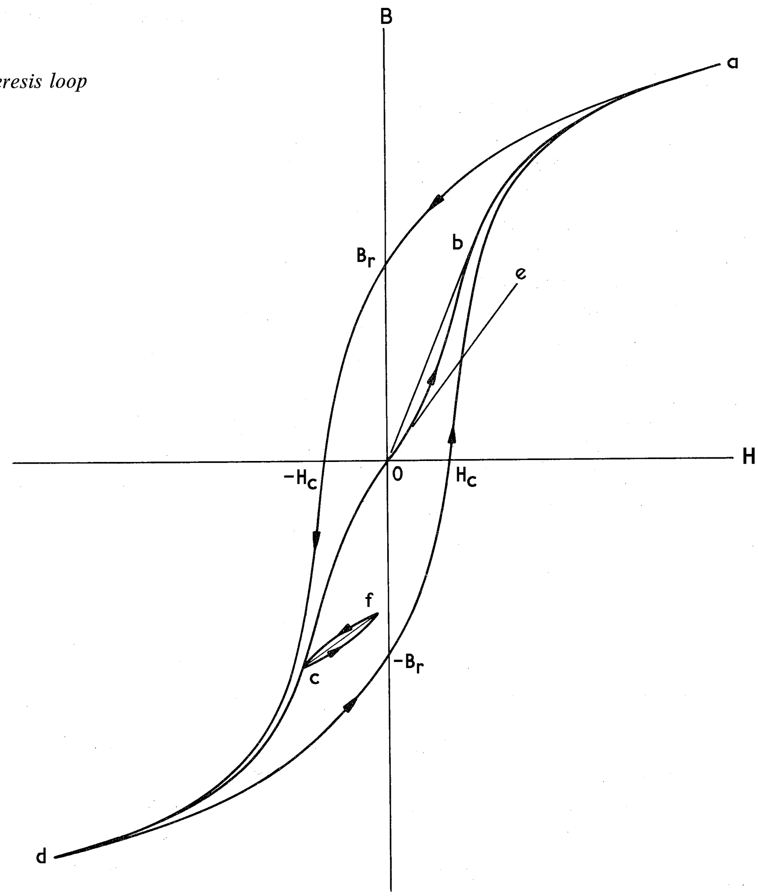

Fig. 2.1. Hysteresis loop

upper limb of the open loop. As explained in Chapter 1, a reason for this lag in the change of flux density is the irreversible movement of the domain walls. If, on reaching point ***d***, symmetrical with ***a***, the field strength is increased again towards its former positive maximum value, the lower limb of the loop will be traced. The loop is called a hysteresis or *B–H* loop. Although, on the first cycle, the loop may not close exactly, a number of cycles will result in a closed loop. If the excursions of H are symmetrical about the origin the material is then said to be in a symmetrical cyclic magnetic state.

If the field strength is large enough to take the material substantially into saturation, i.e. to a point where the magnetization M cannot be significantly increased, then the intercept of the hysteresis loop with the B-axis, B_r (or $-B_r$), is referred to as the remanence of the material and the intercept with the H-axis, H_c (or $-H_c$), is referred to as the coercivity. The tips of loops for smaller excursions of H lie very close to the initial magnetization curve ***oba***.

Since B is a two-valued function of H, the instantaneous ratio B/H depends on the magnetic history. However in alternating magnetization it is usually relevant only to consider the peak amplitudes of B and H, i.e. the tips of the loops. If the material is in a symmetrical cyclic state and H is vanishingly small, the permeability is designated μ_i, the initial permeability. It is $1/\mu_o$ times the slope of the line *oe*. If H is not vanishingly small, then the permeability is referred to as the amplitude permeability designated μ_a. It is $1/\mu_o$ times the slope of the line connecting the origin to the tip of the loop produced by that particular value of H. As H increases, μ_a increases, until the tip of the loop reaches ***b*** and the slope of the line ***ob*** is the maximum value of $\mu_o\mu_a$.

A non-symmetrical or minor loop is traced if, on reaching a point such as ***c***, the field variation is reversed and the material is cycled between ***c*** and ***f***. The slope of ***cf*** divided by μ_o is called the incremental permeability, μ_Δ, and if the amplitude of the excursion is made vanishingly small it becomes the reversible permeability, μ_r. Finally, the slope at any point on a hysteresis loop or curve is referred to as the differential permeability.

In principle B, H and μ have meaning and may be observed without the need for windings, e.g. in a waveguide or a cavity. However, in the present context, it is by means of windings on magnetic cores that magnetic

This may be derived more convincingly by considering the derivation of Eqn 2.78 and replacing the Peterson coefficient, which depends on μ, by the Legg coefficient, which does not. Using the relation given in Table 2.1, Eqn 2.76 becomes

$$\hat{B}_{3a} = \frac{a}{2\pi}\frac{\mu_0^2\mu^3\hat{H}_a^2}{5}$$

If the magnetic circuit has an effective permeability μ_e, then this may be substituted for μ. It then follows that

$$\frac{\hat{B}_{3a}}{\hat{B}_a} = \frac{a}{2\pi}\frac{\mu_e\hat{B}_a}{5}$$

or

$$\frac{\hat{E}_{3a}}{\hat{E}_a} = \frac{-}{5}\cdot\frac{a\mu\hat{B}_a}{2\pi}\cdot\frac{\mu_e}{\mu}$$

$$= 0{\cdot}6 \tan\delta_h\frac{\mu_e}{\mu} \qquad \text{from Eqn 2·68}$$

The other intermodulation amplitude ratios expressed in terms of $\tan\delta_h$ in Table 2.2 may be similarly converted.

The derivation of the relative magnitudes of the distortion and intermodulation e.m.f's in Section 2.2.7 was based on the assumption that the current wave is sinusoidal. However, the magnitude of the distortion e.m.f.

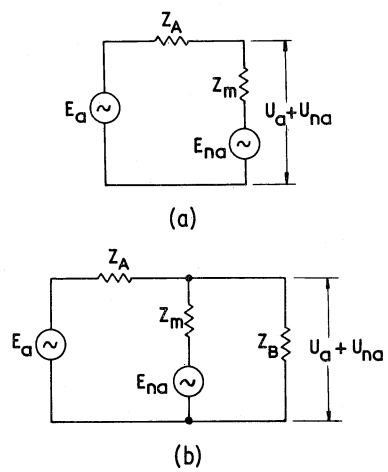

Fig. 4.6. Distortion voltage across the terminals of a loaded inductor or transformer

will be substantially the same even if the applied voltage waveform is sinusoidal, for although, the resulting current waveform will be non-sinusoidal, the amount of distortion considered in an analysis confined to the Rayleigh region is so small that the error in neglecting it will be of second order.

Referring to Fig. 4.6(*a*), Z_m represents the impedance due to the magnetic material and it has in series with it a distortion generator, E_{na}, where n represents the order of the distortion product. The impedance Z_A represents the source impedance, the winding resistance being assumed negligible. If $Z_A \to \infty$, the situation is in accordance with the analysis in Section 2.2.7. The sinusoidal source generator, E_a, will drive a sinusoidal current at frequency f_a through the inductor and the full distortion e.m.f. will appear across the inductor terminals together with the fundamental voltage, U_a. Therefore the distortion ratio $E_{na}/U_a = U_{na}/U_a$ may be measured across the terminals of the inductor.

If Z_A is not infinite, the distortion generator will not have an open-circuit and a distortion current, I_{na}, will flow.

$$I_{na} = \frac{E_{na}}{Z_A+Z_m}$$

where Z_A and Z_m are the impedances observed at the distortion frequency.

Assuming that the amplitude of the fundamental e.m.f. across the inductor is unchanged and the winding resistance is negligible, the distortion voltage ratio across the terminals will be

$$\frac{U_{na}}{U_a} = \frac{E_{na}-I_{na}Z_m}{U_a}$$

$$= \frac{E_{na}}{U_a}\frac{Z_a}{Z_A+Z_m} \qquad (4.58)$$

If, as in the low frequency equivalent circuit of a transformer (see Fig. 7.3), there is also a load impedance, the circuit appears as in Fig. 4.6(b). In this case

$$\frac{U_{na}}{U_a} = \frac{E_{na}}{U_a}\frac{Z}{Z+Z_m} \qquad (4.59)$$

where $Z = Z_AZ_B/(Z_A+Z_B)$, all impedances corresponding to the distortion frequency.

In a well designed transformer Z_m is usually much greater than Z at the distortion frequencies so the distortion voltage ratio will be much less than the distortion e.m.f. ratio.

4.3. OPEN MAGNETIC CORES

4.3.1. General

The magnetic cores considered so far have had either no air gaps or air gaps so small that the flux could be assumed approximately constant round the magnetic path, i.e. the lines of flux leave the magnetic material mainly at the surfaces forming the air gap. When the gap becomes an appreciable fraction of the total magnetic path length, the greater reluctance of the gap causes the flux to leave the magnetic material before crossing the ends of the core which form the gap and the flux is not constant within the core. This invalidates the foregoing treatment and calls

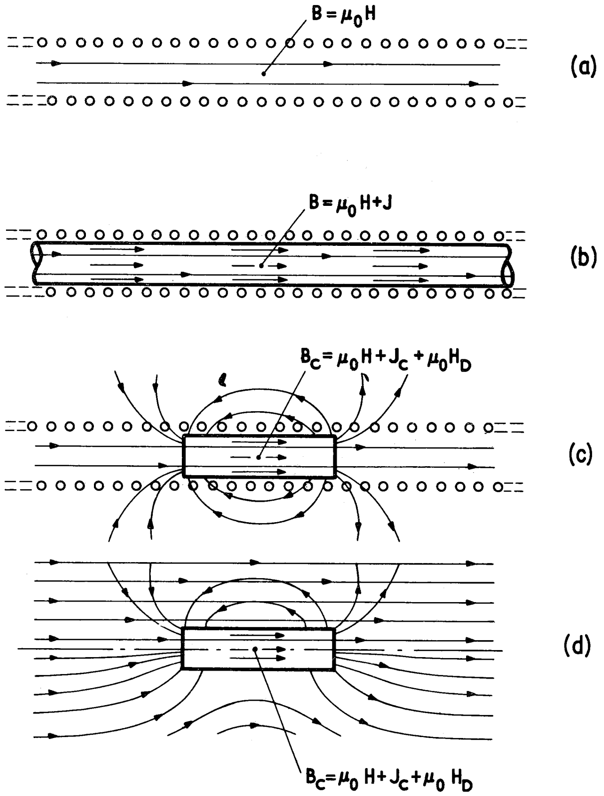

Fig. 4.7. Magnetic flux associated with: (a) a very long air-cored solenoid; (b) a very long solenoid enclosing a very long ferromagnetic cylinder; (c) a very long solenoid enclosing a short cylindrical core; (d) a short cylindrical core immersed in a uniform magnetic field. (upper half: component fields, lower half: resultant field)

for an approach which takes this fringing or leakage flux into account.

The most common form of ferrite core having an air gap large enough to cause appreciable leakage flux is a simple cylindrical or rod core and this section will be mainly concerned with this shape.

Fig. 4.7(*a*) represents an infinitely long solenoid containing no magnetic material. The solenoid carries a current I so the internal field strength is NI/l. The flux density inside the solenoid, represented by the long arrows, is $B = \mu_o H$ and the flux density at all points outside is zero.

The next diagram (*b*) shows the same solenoid containing a very long magnetic core. In addition to the applied field H there is now a field due to the alignment of the magnetic moments of the atomic currents. In a ferromagnetic or ferrimagnetic material the net effect of these atomic currents is to enhance the applied field giving rise to an increased flux density represented by the short arrows. The total flux density in the solenoid is now

$$B = \mu_o H + J \qquad \text{T} \tag{4.60}$$

where J is the magnetic polarization or intrinsic magnetic flux density in tesla (weber. m^{-2}), see Section 2.1. The exterior flux density is still zero because the fields due to the atomic currents cancel at all points outside the core.

If all but a short centre section of the core is removed it is clear that the total field acting in the remaining section will be diminished, since the atomic fields of the removed portions no longer contribute, (see Fig. 4.7*c*). This reduction may be considered to be due to a reverse or demagnetizing field which has neutralized the atomic fields that were previously there. The moments of the atomic currents no longer cancel in their effect outside the solenoid and so an exterior or leakage field exists. Within the remaining portion of the cylindrical core the resultant field will, in general, vary from a maximum in the centre to a minimum at the ends. This may be regarded as due to a non-uniform distribution of the demagnetizing field. In the special case of the remaining portion being an elipsoid the internal field and flux density are constant, and the demagnetizing field is constant.

The demagnetizing field at the centre of the core may be expressed as

$$H_D = -NJ_c/\mu_o = -NM_c \qquad \text{A.m}^{-1} \tag{4.61}$$

where J_c is the magnetic polarization at the centre of the core and N is called the demagnetizing factor. This factor is considered in more detail later. The flux density in the centre of the core is now given by

$$B_c = \mu_o H + J_c + \mu_o H_D \tag{4.62}$$

$$= J_c + \mu_o \left(H - \frac{NJ_c}{\mu_o} \right) \qquad \text{T} \tag{4.63}$$

from 4.61

$$\therefore \quad \frac{B_c - J_c}{J_c} = \frac{\mu_o H_c}{B_c - \mu_o H_c} = \frac{\mu_o H}{B_c - \mu_o H_c} - N$$

(4.60) $B = H + 4\pi M$ Gs

where M is the intensity of magnetization or magnetic moment per cm^3

(4.61) $H_D = -NM_c$ Oe

Note: $(N)_{CGS} = 4\pi(N)_{SI}$

(4.62) $B_c = H + 4\pi M_c + H_D$ Gs

(4.63) $B_c = 4\pi M + H + NM_c$ Gs

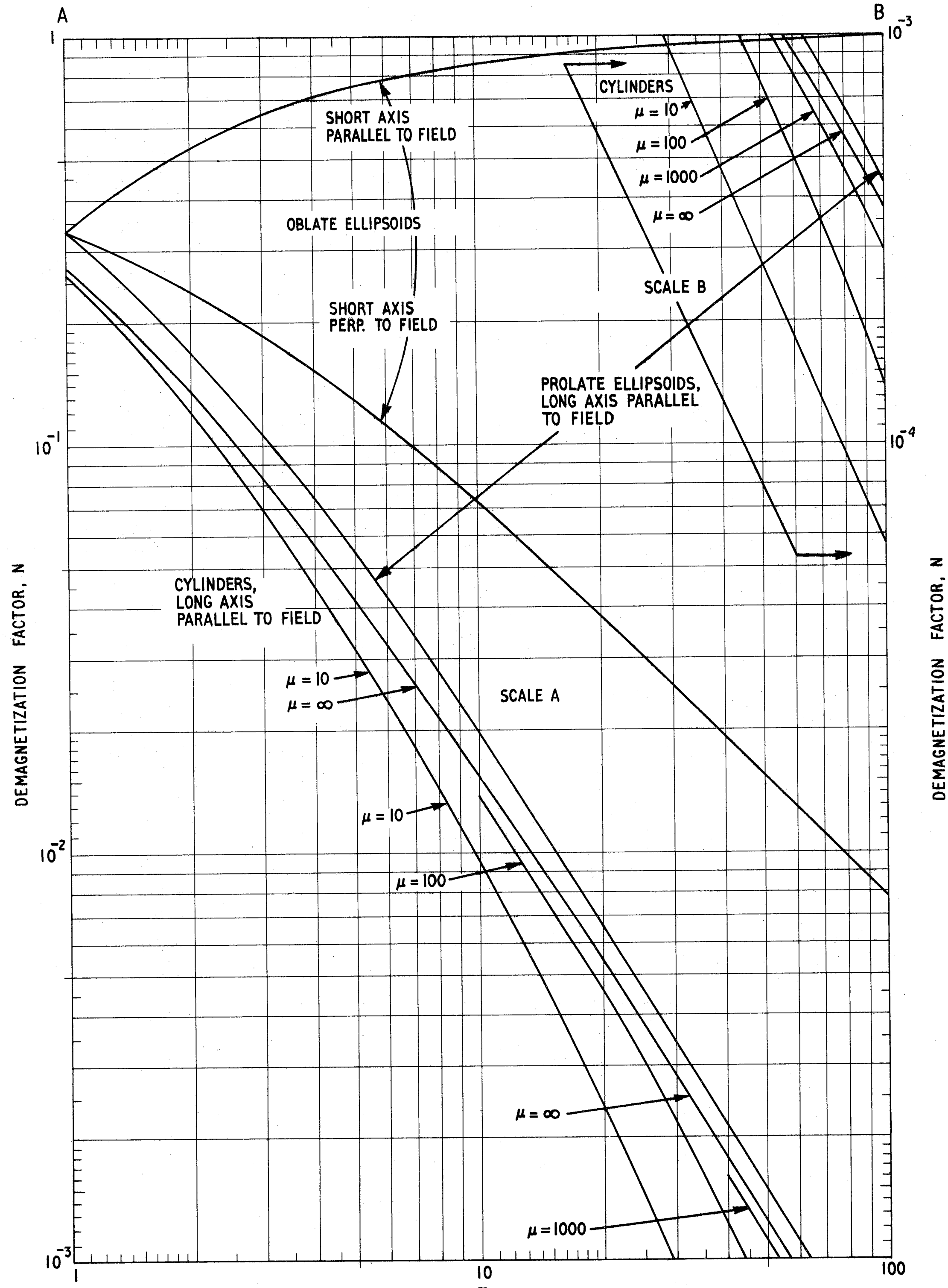

Fig. 4.8. Demagnetizing factors for ellipsoids and cylinders as functions of m (= long axis/short axis for ellipsoids and length/diameter for cylinders) (After Bozorth et al.[6]) Note: The values of **N** *are appropriate to S.I. units;* $(N)_{SI} = 1/4\pi(N)_{CGS}$

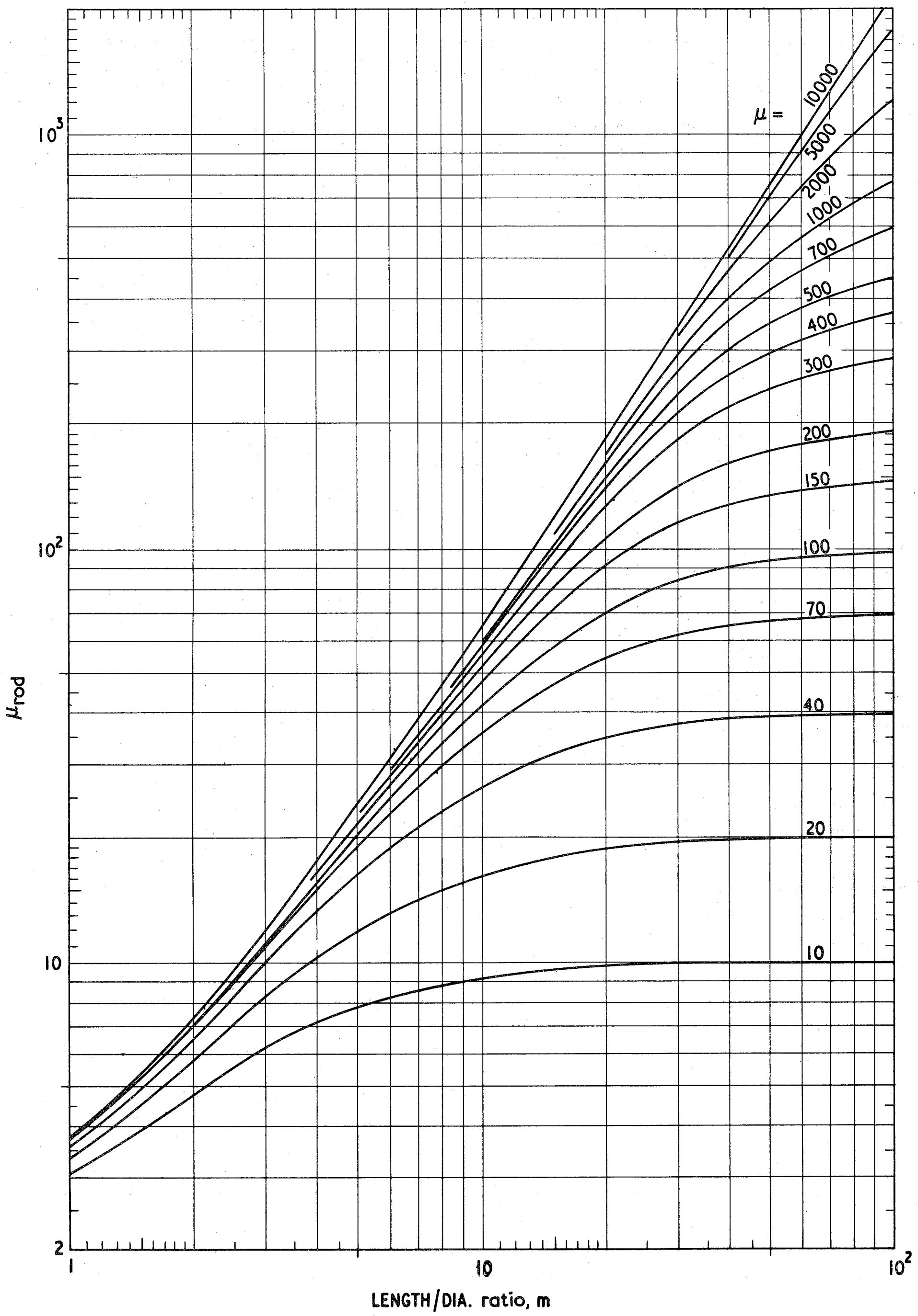

Fig. 4.9. μ_{rod} *as a function of m with material permeability as a parameter*

since the magnetic polarization at the centre, $J_c = B_c - \mu_o H_c$ from Eqn 4.60, where H_c $(= H - NJ_c/\mu_o)$ is the actual field strength at the centre of the rod. The permeability of the material is $\mu = B_c/\mu_o H_c$, while the overall permeability is denoted by μ_{rod} and is defined by $\mu_{rod} = B_c/\mu_o H$ (H being the applied field).

Simplifying the above equation

$$\frac{1}{\mu-1} = \frac{1}{\mu_{rod}}\left(\frac{1}{1-1/\mu}\right) - N \qquad (4.64)$$

Since μ is nearly always much greater than unity, this equation simplifies to

$$\frac{1}{\mu} \approx \frac{1}{\mu_{rod}} - N \qquad (4.65)$$

So far μ_{rod} has been defined as the ratio of the flux density at the centre of the cylindrical core in Fig. 4.7(*c*) to the flux density in the centre of the solenoid in Fig. 4.7(*a*). If, instead of being enclosed by a long close-fitting solenoid, the short cylindrical core were introduced into a relatively large region in which there existed, before the presence of the core, a uniform field H, then the situation would be similar to that depicted in Fig. 4.7(*c*) except that the field, H, would no longer be confined to the interior of the solenoid but would occupy the whole region. It would combine with the leakage field to give a resultant field distribution as shown in Fig. 4.7(*d*). The upper part of this figure shows the component fields while the lower half shows the resultant field. The value of μ_{rod} derived above may now be given an additional definition; it is the ratio of the flux density at the centre of a cylindrical core aligned in a uniform field, to the flux density existing there in the absence of the core. μ_{rod} thus differs from μ_e, the latter referring to a gapped core in which the total flux does not vary significantly along the magnetic path length.

The value of the demagnetizing factor N depends on the geometry of the core and to a lesser extent on its permeability. Fig. 4.8 gives values for cylinders and ellipsoids of revolution. These have been calculated from formulae derived in the literature.[6,7,8] It will be noted that the demagnetization factors of the ellipsoids do not depend on the material permeability. For any body, the sum of the demagnetization factors, relating to three orthogonal axes of that body, is unity. Thus all the ellipsoid demagnetizing factors approach 1/3 as the ellipsoid shape approaches that of a sphere. The demagnetization factors of the cylinders depend on both the dimensional ratio, m (which in this case equals length/diameter), and also on the material permeability.

Using some of these data in Eqn 4.65, μ_{rod} has been calculated as a function of length/diameter ratio for cylinders, with the material permeability as a parameter. The results are shown in Fig. 4.9. This graph shows that when the material permeability is low, the value of μ_{rod} is asymptotic to the material permeability as the rod becomes more slender. This is because the demagnetizing factor becomes very small; from another point of view it could be said that the effective air gap becomes very small. When the material permeability is high the demagnetizing factor or the effective air gap does not become negligible within the practical range of slenderness considered. Even so, the graph shows that rod permeabilities of up to 200 may easily be obtained with practical ferrite rods.

If the ferrite core is a tube having the same material permeability and the same outside dimensions as a rod, then μ_{rod} will be the same, i.e. the flux density in the ferrite half way between the ends of the tube will be $\mu_o . \mu_{rod}$ times the field strength which would exist there in the absence of core. However, the total flux passing through the centre portion of the core would be less than that for a solid rod by the ratio of the cross-sectional areas.

4.3.2. Flux distribution along a cylinder immersed in a uniform magnetic field

It has been seen that when a short cylindrical core is immersed in a uniform field, the flux density varies along the length. The distribution depends on the dimensional ratio of the core and on the permeability. It has been calculated by Warmuth[8] for cores of infinite permeability.

Fig. 4.10(a) shows the measured distribution for a number of cylinders representing typical combinations of permeability and dimensional ratio. Three types of distribution may be distinguished:

(a) Magnetically long cylinders, i.e. m large enough to make $\mu_{rod} \rightarrow \mu$. This gives a rather flattened curve which falls to a low value at the ends of the rod. The lower the material permeability or the higher the value of m the flatter the distribution.

(b) Intermediate cylinders, i.e. m such that μ_{rod} is less than, say, $0{\cdot}8\mu$. An approximately parabolic distribution is obtained. This distribution is similar to that calculated by Warmuth for a cylinder of infinite permeability. If the rod is geometrically short, i.e. such that $m \rightarrow 3$ or less, then the next result applies even if $\mu_{rod} \rightarrow \mu$.

(c) Geometrically short cylinders, i.e. where $m \rightarrow 1$. This distribution is approximately parabolic but is shallower than in (b) i.e. it gives a relatively high

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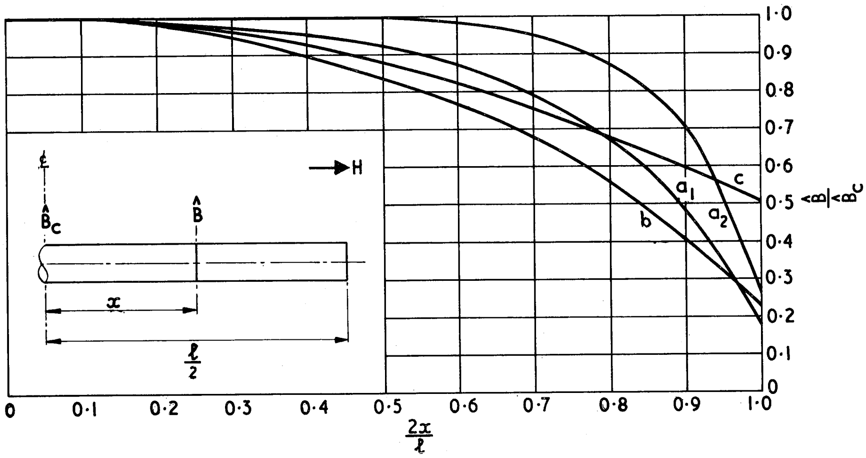
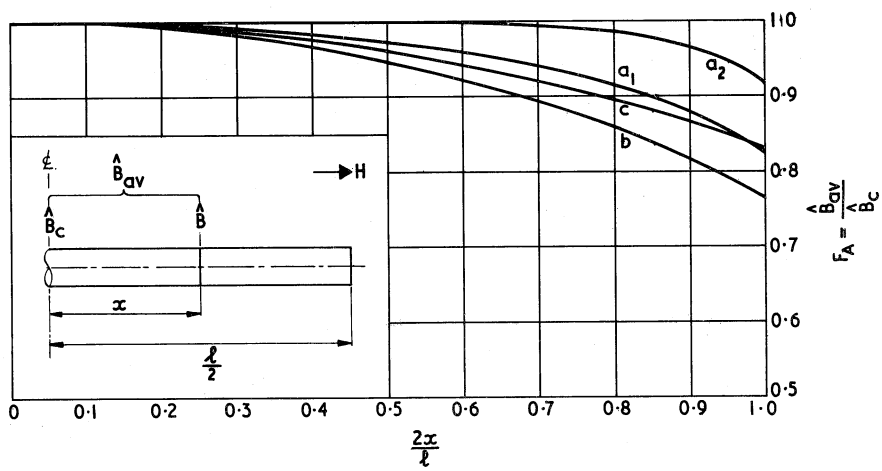
(4.64) $\dfrac{1}{\mu-1} = \dfrac{1}{\mu_{rod}}\left(\dfrac{1}{1-1/\mu}\right) - \dfrac{N}{4\pi}$

(4.65) $\dfrac{1}{\mu} \approx \dfrac{1}{\mu_{rod}} - \dfrac{N}{4\pi}$



(a) Flux density distribution as a function of fractional distance from centre



(b) E.m.f. averaging factor, F_A, as a function of the averaging length, centrally located

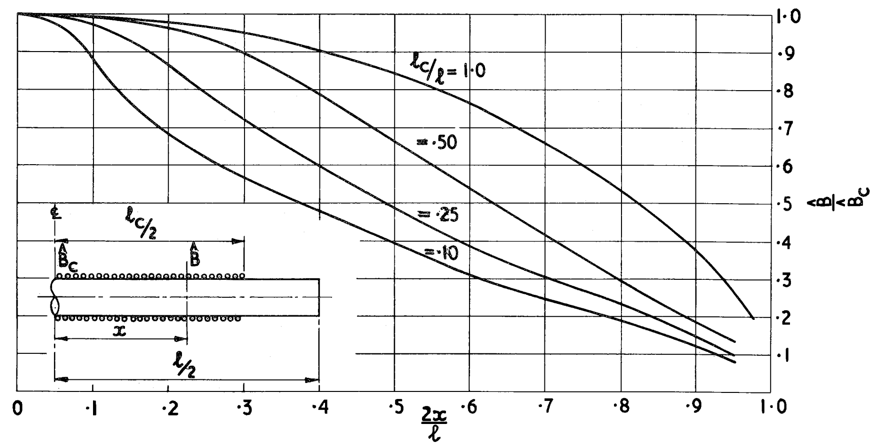
Fig. 4.10. Distribution of flux density measured along various types of ferrite cylinders immersed in a uniform field

	m	μ_i	μ_{rod}	Remarks
a_1	80	50	49·5	Magnetically long,
a_2	80	180	172	$\mu_{rod} \rightarrow \mu_i$
b	10	1000	62	intermediate
	15	1000	114	
c	3	50–1000	10·5–12	Physically short i.e. $m < 5$

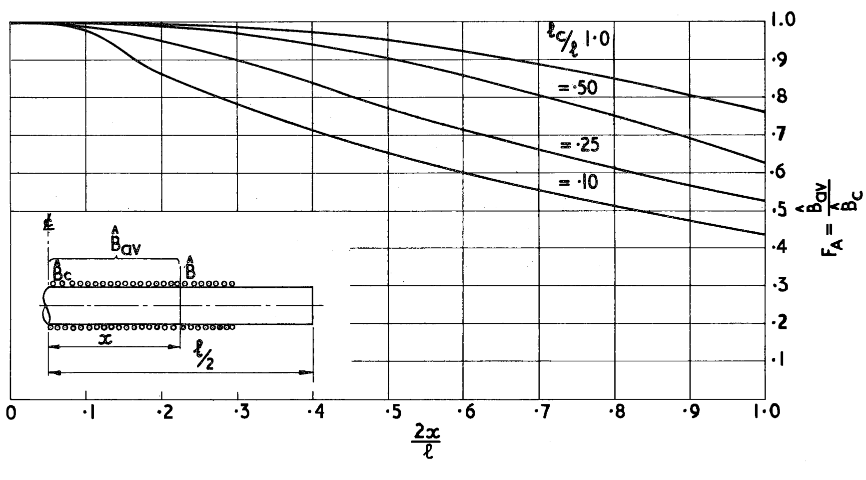
value of flux density at the ends of the rod. In the limit as $m \to 0$ clearly the flux density will become uniform along the axis.

Any actual example may be identified by its value of μ and m as corresponding to, or lying between these types. When the field becomes very large the cylinder may approach saturation at the centre and the permeability may vary from a low value at the centre to a high value at the ends. This tends to make the flux density more uniform over the centre region and the distribution approaches that of (a) above.

A short coil placed in the centre of a rod, will have an e.m.f., E_c, induced in it corresponding to the central flux density $\hat{B}_c$. If the length of the coil is now increased without altering the number of turns the e.m.f. will fall since it will correspond to the average flux density in the part of the rod covered by the coil. The ratio of this e.m.f. to the centre e.m.f., $E/E_c = \hat{B}$ averaged over the length of coil divided by $\hat{B}_c$. This ratio is called the e.m.f. averaging factor, F_A. This factor is given in Fig. 4.10(b) as a function of the fraction of the rod covered by the coil, for the three distributions distinguished above. From this graph the approximate value of F_A may be found for any type of ferrite cylinder immersed in a uniform field.



(a) Flux distribution as a function of fractional distance from centre



(b) E.m.f. averaging factor, F_A, as a function of the averaging length, centrally heated

Fig. 4.11. Distribution of flux density measured along a ferrite cylinder energized by a central solenoid, the parameter being the fraction of the cylinder covered by the solenoid. The result is almost independent of μ_{rod}